

Optically generated 2-dimensional photonic cluster state from coupled quantum dots

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We propose a method to generate a two-dimensional cluster state of polarization encoded photonic qubits from two coupled quantum dot emitters. We combine the recent proposal [5] for generating 1-dimensional cluster state strings from a single dot, with a new proposal for an optically induced conditional phase (CZ) gate between the two quantum dots. The entanglement between the two quantum dots translates to entanglement between the two photonic cluster state strings. Further inter-pair coupling of the quantum dots using cavities and waveguides can lead to a 2-dimensional cluster sheet. Detailed analysis of errors indicates that our proposal is feasible with current technology. Crucially, the emitted photons need not have identical frequencies, and so there are no constraints on the resonance energies for the quantum dots, a standard problem for such sources.

Measurement-based quantum computation (MQC) is an alternative to the well-known ‘circuit model’ of quantum computation [1]. The main idea in MQC is to robustly create, upfront, a highly entangled state. Once this ‘cluster state’ is created, which is the challenging part of this approach, only single qubit measurements are necessary to perform the actual computation. In the case of photon polarization qubits, performing single qubit rotations followed by photon number detection is easily done with high fidelity, which makes them particularly attractive for MQC. In fact this is one of the most fault-tolerant architectures known for quantum computing [2], and is particularly tolerant to qubit losses [3], of importance for optical architectures. The creation of the initial entangled cluster state is, however, a difficult problem on which much current research efforts are focused. To date the most promising methods have involve optical interference of nearly identical photons [4]. By contrast, our proposal here allows for direct generation of the entangled photons.

In Ref. [5] a proposal was developed for generating a linear (one-dimensional) cluster state of polarization encoded photons from single photon emitters with a certain energy level structure, such as those found in quantum dots (QDs). The relevant states of the QD are the two spin states $|\uparrow\rangle, |\downarrow\rangle$ of the electron along the optical axis z and the two optically excited states called trions, which have total angular momentum $3/2$ and have spin projections along the z -direction of $\pm 3/2$ - states we denote $|3/2\rangle, |\bar{3}/2\rangle$. The broken symmetry of the QD along the z axis sets a preferred direction, along which the optical polarization selection rules are circularly polarized, and energetically separates the excited trion states with total angular momentum $\pm 1/2$ (the light hole states) from these heavy-hole trion states. In the process of linear cluster state generation [5] the heavy hole trions are the only excited states that are populated. The main idea in [5] is to shine a periodic train of optical linearly po-

larized π pulses, to an electron that is in a superposition state $|\uparrow\rangle + |\downarrow\rangle$, exciting it to a superposition of the two trion states $|3/2\rangle + |\bar{3}/2\rangle$. Because QDs have large dipole moments, spontaneous emission is very fast, both compared to atoms and to the other relevant time scales in the QD dynamics, at least for very low magnetic fields. Therefore the trion will spontaneously decay to the electron state almost instantaneously upon excitation, emitting a photon of either right (R) or left (L) circular polarization, thereby effecting transitions $|3/2\rangle \rightarrow |\uparrow\rangle|R\rangle, |\bar{3}/2\rangle \rightarrow |\downarrow\rangle|L\rangle$. The state of the emitted photon+spin is $|\uparrow\rangle|R\rangle + |\downarrow\rangle|L\rangle$ - i.e. they are entangled as both recombination paths take place simultaneously. The remaining degrees of freedom of the system are the same, so they are factored out and omitted for brevity. Subsequent precession of $\pi/2$ radians by the spin about a weak magnetic field oriented in the y -direction is performed, denoted $R_y(\pi/2)$, before subjecting the dot to another pulse excitation+emission process. Repeating this protocol results in a one-dimensional entangled chains of photons. Importantly, errors were shown to localize and not affect the whole chain.

Here we will develop an explicit, all-optical protocol for generating a two-dimensional cluster state comprised by linking two linear chains like the ones of [5] by controlled phase (CZ) gates. To do so we present a new proposal, related to that of [6], for performing an optically-controlled CZ gate between two quantum dots. Taking advantage of the exchange interactions between electrons and the hole, this gate actually proves to be faster than that of [6] and so is of independent interest. Crucially, unlike the scheme of [6], this process is also compatible with the operation of the single-dot photonic machine guns, as it performs the optical CZ in the z basis. The entangled emitters therefore generate photons which are themselves entangled. Explicitly, an entangled chain can be created. This circumvents the need for ‘fusion gates’ [4]. Moreover, in our approach, the photons need not be identical in fre-

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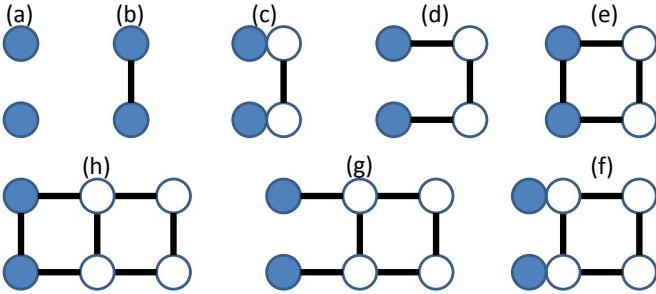


FIG. 1: A sequence of diagrams depicting the generation of the cluster state using the standard diagrammatic representations of such states. The electronic qubits are depicted as filled circles, the initial electronic state is taken to be $|\uparrow\rangle\langle\uparrow|$. At step (a) both spins precess under $R_y(\pi/2)$, at (b) the interdot CZ gate is applied, at (c) a pulse excitation followed by trion decay produces photons (open circles). These procedures are then repeated, leading to the states of (d), ..., (h). Details of the states produced are in the text. Note that to recover the standard form of cluster states one must use a mapping where the logical qubit $|1\rangle$ state is equivalent to the photonic state $-|L\rangle$, this is because, for practical reasons, $R_y(\pi/2)$ gates are used instead of Hadamard gates.

quency, so that there are no constraints on the resonance energies of the two QDs.

The state evolution for the idealized abstract protocol is depicted in Fig. 1, for a quantum circuit logically equivalent to the protocol see Fig. 2. For simplicity we assume that the two QDs are initialized in the spin up state $|\uparrow\rangle\langle\uparrow|$ (in fact no initialization is necessary - it can be effected later via measurements on the photons). First we apply a $R_y(\pi/2)$ operation on each spin yielding $(|\uparrow\rangle + |\downarrow\rangle)(|\uparrow\rangle + |\downarrow\rangle)$, as in Fig. 1(a). This is followed by a C-Z gate entangling the dots, $(|\uparrow\rangle\langle\uparrow| + |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|)$, producing the bond in Fig. 1(b). Immediately after this we apply the pump pulse to each dot, and the creation of the subsequent photons yield the state $(|\uparrow\rangle|R\rangle\langle\uparrow|R\rangle + |\uparrow\rangle|R\rangle\langle\downarrow|L\rangle + |\downarrow\rangle|L\rangle\langle\uparrow|R\rangle - |\downarrow\rangle|L\rangle\langle\downarrow|L\rangle)$. In the circuit of Fig. 2 this is equivalent to the CNOT gates. This resulting state is equivalent to a 2-qubit cluster state, where the logical state $|0\rangle$ ($|1\rangle$) is redundantly encoded [4] in 2 qubits as $|\uparrow\rangle|R\rangle$ ($-|\downarrow\rangle|L\rangle$). Graphically such a situation is depicted with the circles for each qubit adjacent to each other, Fig. 1(c). A second $R_y(\pi/2)$ on each dot pushes out the redundantly encoded qubits (i.e., creates a bond between them in the cluster state), Fig. 1 (d), and we start the cycle anew.

The inter-dot C-Z gate is implemented optically by coupling to trion states which are higher in energy than the ones used for the single dot photon emission. These higher-energy trion states are delocalized, i.e. the volt-

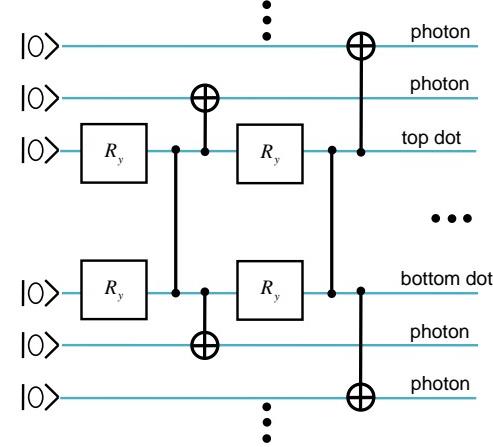


FIG. 2: A quantum circuit which is logically equivalent to the idealized evolution of the two QDs. The CZ gates correspond to the interdot coupling, the CNOT gates to photon emission, and the R_y to precession by $\pi/2$ around a magnetic field in the y direction.

age bias is such that one of the electrons in these higher energy states is tunnel-coupled, in contrast to the single-electron ground states (denoted $|B\rangle$ and $|T\rangle$ for bottom and top QD respectively) and the lower energy trion states, which are isolated from one another and localized to their respective quantum dots. This trion mediating the inter-dot interaction has two electrons in the $|B\rangle$ and $|T\rangle$ ("s"-type) states, and the third electron in the first excited ("p"-type) orbital, which we take to be the one that is a delocalized (also called 'molecular' or 'extended') state, denoted $|E\rangle$. It has been shown experimentally that such a regime is feasible [7]. The hole is taken to occupy a single orbital state $|H\rangle$, which for simplicity we take to be completely confined to one QD (this assumption does not deleteriously affect the overall proposal).

With the three electrons in distinct orbital states, the spin configuration can acquire any of its allowed values by adding the three angular momenta. So, for a given orbital configuration there are a total of eight electron states (two $S = 1/2$ doublets and one $S = 3/2$ quadruplet) and two hole spin states, making a total of sixteen states [6]. By tuning the laser appropriately, some of the states can be safely ignored due to the large e-e exchange splittings (on the order of 5 meV) compared to the laser bandwidth.

We henceforth focus on the $S = 3/2$ states, which have a separable (product) form of orbital and spin states:

$$|3/2\rangle = |A\rangle|\uparrow\uparrow\uparrow\rangle \quad (1)$$

$$|1/2\rangle = |A\rangle(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle)/\sqrt{3} \quad (2)$$

$$|\bar{1}/2\rangle = |A\rangle (|\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\rangle) / \sqrt{3} \quad (3)$$

$$|\bar{3}/2\rangle = |A\rangle |\downarrow\downarrow\downarrow\rangle \quad (4)$$

where

$$\begin{aligned} |A\rangle &= \frac{1}{\sqrt{6}} (|TBE\rangle - |BTE\rangle - |TEB\rangle \\ &\quad + |BET\rangle - |EBT\rangle + |ETB\rangle). \end{aligned} \quad (5)$$

The electron and hole in semiconductors are coupled by exchange interactions. In QDs, these are quite strong (on the order of, or stronger, than typical Zeeman energies), and they are separable into ‘isotropic’ and ‘anisotropic’ terms [8]. The isotropic term is much stronger - typical values of this are 0.3–0.5 meV - so it is the leading term in our parameter regime. Its physical origin is the lack of inversion symmetry (along the growth direction) in the QD. We will ignore the anisotropic term, which originates from in-plane asymmetry (deviation of the QD cross section from a disk) and is typically small, in the order of μeV [8, 9]; its effects can be incorporated as standard errors in the gate.

The Hamiltonian is therefore given by $\mathcal{H} = \sum_i \alpha_z(r_i, r_h) s_{iz} j_z$, where $\alpha_z(r_i, r_h)$ is an operator acting on the envelope wavefunctions of the electrons and the hole. The index i runs over the three electrons, r_h denotes the position of the hole, and z is the growth axis. The operator j acts on the hole spin, which we take to be a pseudospin, $j_z = \pm 3/2$ (i.e., ignore light hole states). Adding and subtracting terms, we can rewrite \mathcal{H} as

$$\begin{aligned} \mathcal{H} &= \frac{1}{3} \sum_i \alpha_z(r_i, r_h) S_z j_z \\ &\quad + \frac{1}{3} \sum_{\substack{(ij) \in \\ \{(12), (23), (31)\}}} (\alpha_z(r_i, r_h) - \alpha_z(r_j, r_h)) (s_{zi} - s_{zj}) j_z, \end{aligned} \quad (6)$$

with $S_z = \sum_i s_{iz}$. The first term on the RHS of Eq. (6) conserves the total electron spin, so it only has nonzero matrix elements within the three total spin subspaces discussed above. The second set of terms has nonzero matrix elements only between different total electron spin states. Since typical values of the electron-electron exchange are about one order of magnitude more than typical electron-hole exchange interactions, we can ignore the total spin mixing terms and focus on the Hamiltonian

$$\mathcal{H} \simeq \frac{1}{3} \sum_i \alpha_z(r_i, r_h) S_z j_z, \quad (7)$$

and only consider the states (1)-(4) tensored with the hole state, which is an 8×8 space. The mean value of the operator α in state $|A\rangle|H\rangle$ is

$$\begin{aligned} &\sum_i \langle H | \langle A | \alpha(r_i, r_h) | H \rangle | A \rangle \\ &= \frac{2 \times 3}{6} \sum_{K=B,T,E} \langle H | \langle K | \alpha(r, r_h) | H \rangle | K \rangle \\ &\equiv \delta_0^{BH} + \delta_0^{TH} + \delta_0^{EH}. \end{aligned} \quad (8)$$

Assuming that the hole is localized in one of the two quantum dots, say the one labeled by B , we have $\delta_0^{TH} = 0$. We will define the sum of the nonzero terms to be δ_0 . Now we have the operator

$$\mathcal{H}_{3/2} = \frac{\delta_0}{3} S_z j_z, \quad (9)$$

acting only on the spin states. Clearly, this operator is already diagonal in the basis we have chosen. Since it is invariant under the simultaneous flip of S_z and j_z , we expect the states to be doubly degenerate. Then the eigenenergies and corresponding eigenstates are:

$$E_1 = \frac{\delta_0}{4} \text{ with eigenstates } |\bar{3}/2\rangle |\uparrow\rangle, |\bar{3}/2\rangle |\downarrow\rangle \quad (10)$$

$$E_2 = \frac{\delta_0}{12} \text{ with eigenstates } |1/2\rangle |\uparrow\rangle, |\bar{1}/2\rangle |\downarrow\rangle \quad (11)$$

$$E_3 = -\frac{\delta_0}{12} \text{ with eigenstates } |1/2\rangle |\downarrow\rangle, |\bar{1}/2\rangle |\uparrow\rangle \quad (12)$$

$$E_4 = -\frac{\delta_0}{4} \text{ with eigenstates } |\bar{3}/2\rangle |\downarrow\rangle, |\bar{3}/2\rangle |\uparrow\rangle \quad (13)$$

The states with energy E_1 are dark. The remaining ones are optically accessible. We are particularly interested in the states with energy E_4 . These states are coupled only to the two-qubit states $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ by polarization σ^- and σ^+ respectively. The two-qubit states $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ couple to the states with E_2, E_3 with these polarizations. We take advantage of the energy splitting between E_4 and E_2, E_3 to selectively address only the two-qubit $|\downarrow\downarrow\rangle$ state and realize the C-Z gate.

For simplicity we fix the polarization of the pulse to σ^+ (behaviour for the orthogonal polarization is found by flipping all the spins). If we label the dipole matrix element for transition $|\downarrow\downarrow\rangle \rightarrow |\bar{3}/2\rangle |\uparrow\rangle$ to be d_0 . Then only the triplet state $|T_+\rangle$ couples to the excited state $|\bar{1}/2\rangle |\uparrow\rangle$ with dipole strength $\sqrt{\frac{2}{3}}d_0$.

Given these three transitions, we can implement the C-Z gate by acting with a resonant 2π pulse on the $|\downarrow\downarrow\rangle$ state and avoid coupling to the other transitions.

We now turn to a consideration of the various sources of errors and imperfections. A crucial feature of our proposal is the fact that all non-leakage errors in the system *localize*. By non-leakage errors we refer to any decoherence which eventually returns the electrons back into the computational subspace - ie back into any state such that one electron is located in the orbital ground state of each dot. By localize we refer to the fact that the action of any decoherence map on the electrons is (mathematically) equivalent to a (different) decoherence map on some of the emitted photons, however crucially the number of affected photons is at most the four photons emitted around the time the decoherence event occurs. This ensures that the final output state takes the form of an ideal cluster subject to localized random noise—a noise model for which fault tolerant procedures are known to work.

In particular we emphasize that this allows for production of photonic cluster states for arbitrarily longer times than the electron decoherence timescales might suggest.

The error localization might be seen in quite a general manner as follows. Consider the quantum circuit of Fig. 2 encoding the generic evolution. Let some decoherence occur which is described by a set of Kraus operators $\{K_i\}$ acting on the spin only. If we denote by U the unitary evolution which corresponds in the figure to the circuit consisting of four photon emissions (i.e., two photons per dot and including the CZ gate acting between the dots) then an error and subsequent evolution takes the generic form

$$\rho'_i = U(I \otimes I \otimes K_i)(\rho_{spin} \otimes |0\rangle\langle 0| \otimes |0\rangle\langle 0|)(I \otimes I \otimes K_i^\dagger)U^\dagger.$$

It is a remarkably nice feature of this process that in fact we can find a Kraus operator \tilde{K}_i acting now only on the four emitted photons, such that

$$\rho'_i = (\tilde{K}_i \otimes I)U(\rho_{spin} \otimes |0\rangle\langle 0| \otimes |0\rangle\langle 0|)U^\dagger(I \otimes \tilde{K}_i^\dagger).$$

Physically this means that an error occurring on the spin is (mathematically) identical to some different error occurring on the photons subsequently emitted. Crucially it affects only the next four photons, and no more—hence the term localization.

We now discuss some specific sources of error, and their expected impact.

1. Imperfect CZ gate—If we label Ω_0 the Rabi frequency of the target transition from $|\downarrow\downarrow\rangle$, then the other transitions see a Rabi frequency of $\Omega_1 = \Omega_0/\sqrt{3}$ and $\Omega_2 = \sqrt{\frac{3}{2}}\Omega_0$, with a large detuning. As such some population is transferred to those excited states and it is not returned via stimulated emission. Instead, the incoherent process of spontaneous emission redistributes that population. For simplicity we assume that the small population transferred is equal for the two unwanted transitions and that spontaneous emission equally redistributes it. The simplest way to express the Kraus operators $\{K_j\}$ describing the generalized quantum evolution in the two spin qubit subspace is by one nearly unitary, CZ operator:

$$K_0 = u_1 |\uparrow\uparrow\rangle\langle\uparrow\uparrow| + |\psi^-\rangle\langle\psi^-| + u_2 |\psi^+\rangle\langle\psi^+| - |\downarrow\downarrow\rangle\langle\downarrow\downarrow|,$$

plus eight more operators describing the redistribution of the populations. For a pulse of a total duration of 40 ps and for anisotropic exchange $\delta = 0.5$ meV we have $|u_1| \simeq |u_2| \sim 0.99$. Then the remaining operators, $\{K_1, K_2, \dots, K_8\}$, are $\frac{\sqrt{1-|u_1|^2}}{2}|k\rangle\langle 1|$ and $\frac{\sqrt{1-|u_2|^2}}{2}|k\rangle\langle 3|$, with $k = 1, 2, 3, 4$. Since the operator sum representation is not unique, we can find a different set of Kraus operators $\{M_j\}$ for which M_0 is proportional to the CZ gate. Setting $u_1 = u_2 \equiv u$, these are $M_0 = \alpha$ CZ, $M_1 = e^{i\phi}\sqrt{1-\alpha^2}K_0 - \frac{\alpha}{\sqrt{2}}(K_1 + K_2)$, and $M_2 = \frac{1}{\sqrt{2}}(K_1 - K_2)$,

with $\phi = \arctan\left(\frac{Im(u)}{1-Re(u)}\right)$. For $j = 3, \dots, 8$, $M_j = K_j$. The value of α is a measure of how close the operation is to a unitary CZ. For $u_1 = u_2 = 0.99$ we find $\alpha = 0.98$, with an error of $1-\alpha = 0.02$. Physically we can therefore interpret the action of the gate as follows: With probability α^2 we obtain a perfect CZ gate, with probability $(1-\alpha^2)$ we obtain some other type of evolution.

2. Unequal g factors—In general, the two QDs comprising the QD molecule will have different g factors, and therefore different precession frequencies. This means that we cannot get both spins to undergo a $R_y(\pi/2)$ operation solely based on precession. One can correct for this mismatch by spin-echo type control by applying to the fast spin at time $\tau = \pi(\omega_f^{-1} - \omega_s^{-1})/4$ a single qubit π rotation about the optical axis to delay it (ω_f, ω_s are the fast and slow Zeeman splittings respectively). These rotations are by design fast (in the ps regime) [10] and have been demonstrated experimentally [11].

3. Decay of one of the two resident electrons into the other QD—When this error occurs it will cause both quantum dots to stop emitting photons, and thus amounts to a detectable loss error on the cluster state. We want the hole to occupy the dot for which the single particle energy is lowest so that recombination will now correct for this error. In general we expect the computation to be quite resilient to such loss. Depending on the height of the tunneling barrier between the dots, eventually the system will decay back into the desired computational basis with one electron in each dot.

4. Precession during CZ gate—This is an error whose effect will be essentially the same as the case of the single dot machine gun[5]. It results in a localizable error, which, provided the magnetic field strength is chosen suitably, can be extremely low.

In conclusion, we have developed a scheme for generation of $2 \times N$ dimensional photonic cluster state based on coupled quantum dots. Analysis of the relevant errors shows our proposal to be robust and feasible with current state of the art systems. This scheme can be generalized to the generation of a two-dimensional sheet either by considering multiple stacked dots, or by employing cavities and waveguides to couple distant dots. Future work will include specific cavity-waveguide-quantum dot designs for generation of a cluster state sheet of arbitrary dimensions.

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- [1] R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. **86**, 5188 (2001).
 - [2] R. Raussendorf *et. al.*, Annals of Physics **321**, 2242 (2006);

- [3] M. Varnava *et. al*, Phys. Rev. Lett. **97**, 120501 (2006); *ibid*, Phys. Rev. Lett., **100**, 060502 (2008).
- [4] D. E. Browne and T. Rudolph Phys. Rev. Lett. **95**, 010501 (2005); Q. Zhang, et al, Phys. Rev. A. **77**, 062316 (2008).
- [5] N. H. Lindner and T. Rudolph, Phys. Rev. Lett. **103**, 113602 (2009).
- [6] S. E. Economou and T. L. Reinecke, Phys. Rev. B **78**, 115306 (2008).
- [7] E. A. Stinaff, M. Scheibner, A. S. Bracker, V. L. Ponomarev, V. L. Korenev, M. E. Ware, M. F. Doty, T. L. Reinecke, and D. Gammon, Science **311**, 636 (2006).
- [8] M. Bayer, G. Ortner, O. Stern, A. Kuther, A. A. Gorbunov, A. Forchel, P. Hawrylak, S. Fafard, K. Hinzer, T. L. Reinecke, et al., Phys. Rev. B **65**, 195315 (2002).
- [9] M. Scheibner *et. al.*, Phys. Rev. B **75**, 245318 (2007)
- [10] S. E. Economou, L. J. Sham, Y. Wu, and D. G. Steel, Phys. Rev. B **74**, 205415 (2006).
- [11] A. Greilich, S. E. Economou, S. Spatzek, D. R. Yakovlev, D. Reuter, A. D. Wieck, T. L. Reinecke, and M. Bayer, Nat. Phys. **5**, 262 (2009).